Formelsammlung

Elektronische Bauelemente

Die meisten Formeln/Zusammenhänge sind dem Buch zur Vorlesung „Elektronische Bauelemente“ von Prof. Dr. rer. nat. Klaus Heime entnommen. Hinter den einzelnen Formeln wird deshalb auf die jeweiligen Seiten im Buch verwiesen.
Diese Formelsammlung wurde 1993 erstellt und seit dem nicht aktualisiert. Nach einer Anfrage im September 2001 habe ich nun die Umlaute angepaßt, Anpassungen an LaTeX2e durchgeführt, meine eMail-Adresse korrigiert und die Formelsammlung inkl. LaTeX-Quellcode ins WWW gestellt. Dort ist sie unter der Adresse http://www.oche.de/ leutloff/elektrotechnik/ abrufbar.

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Letzte Änderung am 15.12.93.

Alle Angaben ohne Gewähr!

1 Potentialstufe und Potentialbarriere

Potentialstufe		
Transmissionsfaktor $E < V_0$	$T = 0$ und damit $R = 1$	
$E > V_0$	$T = \frac{4k_{II}}{k_I(1+\frac{k_{II}}{k_I})^2}$	(2.46)
	mit $k_I = \sqrt{\frac{2mE}{\hbar^2}}$ und $k_{II} = \sqrt{\frac{2m(E-V_0)}{\hbar^2}}$	(2.42)
Reflexionsfaktor	$R = 1 - T$	(2.45)
Potentialbarriere (Breite a)		
Transmissionsfaktor $E < V_0$ (Tunneleffekt)	$T = \left[1 + \frac{\sinh^2\left(\frac{a}{\hbar}\sqrt{2m(V_0-E)}\right)}{4E(V_0-E)}\right]^{-1}$	(2.55)
$E > V_0$	$T = \left[1 + \frac{\sinh^2\left(\frac{a}{\hbar}\sqrt{2m(E-V_0)}\right)}{4E(E-V_0)}\right]^{-1}$	(2.55)
Reflektionsfaktor	$R = 1 - T$	
Antreffwahrscheinlichkeit/Eindringtiefe (langer Stab)	$\frac{dE(x)}{dx} \approx e^{-2k_{II}\,x}$ mit $k_{II} = \sqrt{\frac{2m(E-V_0)}{\hbar^2}}$ $\Rightarrow$ Eindringtiefe $x_E = \frac{1}{2k_{II}}$	
DE BROGLIE-Wellenlänge	$\lambda = \frac{h}{\sqrt{2mE}}$	(2.47) (2.130)

2 Bändermodell für in- und extrinsische Halbleiter

(für intrinsische Halbleiter wird E_F durch E_{Fi} ersetzt) $\boxed{\boxed{\text{n(T) 3.84}}}$

Besetzungswahrscheinlichkeit	$f(E) = \frac{1}{1 + e^{\frac{E-E_F}{kT}}}$	(3.27)
effektive Zustandsdichten	$N_L = 2\left(\frac{2\pi\, m_n^*\, kT}{h^2}\right)^{\frac{3}{2}}$ $\quad N_V = 2\left(\frac{2\pi\, m_p^*\, kT}{h^2}\right)^{\frac{3}{2}}$	(3.44) (3.47) (3.75)
Konzentration	davon besetzt: $n = N_L\, e^{-\frac{E_L-E_F}{kT}}$ $\quad p = N_V\, e^{-\frac{E_F-E_V}{kT}}$	
	$N_V = N_L \left(\frac{m_p^*}{m_n^*}\right)^{\frac{3}{2}}$ $\quad \frac{E_g}{2} = E_{Fi} - E_V$	
Konzentration der Teilchen im Reservebereich eines extrinsischen Halbleiters (Aktivierungsenergie: ΔE_D, ΔE_A)	$n = \sqrt{N_D N_L}\, e^{-\frac{\Delta E_D}{2kT}}$ $\quad p = \sqrt{N_A N_L}\, e^{-\frac{\Delta E_A}{2kT}}$	(3.83)
Massenwirkungsgesetz	$n_i^2 = np = N_L N_V\, e^{-\frac{E_L-E_V}{kT}}$ $E_L = E_V + E_g$ im intrinsischen Halbleiter ist $n = p = n_i$	(3.47) (3.75)
Fermi-Niveau im intrinsischen Halbleiter	$E_{Fi} = \underbrace{E_V + \frac{1}{2}\, E_g}_{E_{Fi}(T=0)} + \underbrace{\frac{3}{4}\, kT\, \ln \frac{m_p^*}{m_n^*}}_{\Delta E_{Fi}(T)}$	(3.49)
Ladungsbilanz im extrinsischen Halbleiter	$n + N_A^- = p + N_D^+$ (Neutralitätsbedingung)	
	$N_D^+ = N_D \left(1 - \frac{1}{1 + e^{\frac{E_D-E_F}{kT}}}\right)$ $N_A^- = N_A \left(\frac{1}{1 + e^{\frac{E_A-E_F}{kT}}}\right)$	(3.79)
	mit $N_D = N_D^0 + N_D^+$ und $N_A = N_A^0 + N_A^-$	

thermische Geschwindigkeit	$v_{th} = \sqrt{\frac{3kT}{m^*}} \qquad E_{kin} = \frac{3}{2}kT$	(3.88)
Beweglichkeit	$\mu_n = \frac{q\tau_n}{m_n^*} \qquad \mu_p = \frac{q\tau_p}{m_p^*}$	(3.93)
Leitfähigkeit	$\sigma = \kappa = q(n\mu_n + p\mu_p) \quad (\ \sigma = qn_i(\mu_n + \mu_p))$	(3.93)
Abstand der Bänder PN-Übergang Diagramm 4.3	$\eta_n = E_{Ln} - E_F = kT\ \ln \frac{N_L}{n_n}$ $\eta_p = E_F - E_{Vp} = kT\ \ln \frac{N_V}{p_p}$	(4.2) (7.81)
EINSTEINsche Beziehung und Diffusionskoeffizient	$D_n = \mu_n U_T \qquad D_p = \mu_p U_T$ mit $U_T = \frac{kT}{q} = 25,9$ mV bei 300 K	(4.10)
	$\vec{S} = -D\ \mathrm{grad}\, n$	(4.8)
Austrittsarbeit	[$q\Phi_{Hl} = E_0 - E_F$!!Vorsicht!! bitte nachlesen!!]	(6.6)
Volumenpotential	$q\Psi_b = E_F - E_{Fi}$	(6.6)
Oberflächenpotential	$\varphi_S = \frac{qN_A^-}{2\varepsilon_0\varepsilon_r}\, l^2$ (> 0 bei Verarmung eines p-Halbl.) $l = \frac{N_S^+}{N_A^-}$ $l = \underbrace{\sqrt{\frac{\varepsilon_0\varepsilon_r U_T}{qN_A^-}}}_{\text{Debye-Länge } L_{Dp}} \sqrt{\frac{2q\varphi_S}{kT}}$	(6.11) (6.12) (6.18)

Jeder weiß:

Energieerhaltungssatz: $h\nu = q\Phi_m + E_{Kin}$
Driftgeschwindigkeit: $\vec{v}_{dn} = -\mu_n\vec{E}; \quad \vec{v}_{dp} = \mu_p\vec{E}$ (3.93)

3 PN-Übergang

(Nicht alle Formeln gelten für den PIN-Übergang!)

Diffusionsspannung	$-qU_D = E_g - \eta_{p0} - \eta_{n0} \qquad U_D = -U_T \ln \frac{N_A N_D}{n_i^2}$	(7.81) (7.91)
maximale Feldstärke	$E_{0max} = \frac{2U_D}{l_0} \qquad \text{für U} = 0$	(7.89)
Ausdehung der Raumladungszone im p-Gebiet	$l_p = \sqrt{\frac{2\varepsilon_0\varepsilon_r N_D}{qN_A(N_A+N_D)}(-U_D-U)}$ [1]	(7.93)
im n-Gebiet	$l_n = \sqrt{\frac{2\varepsilon_0\varepsilon_r N_A}{qN_D(N_A+N_D)}(-U_D-U)}$ [1]	(7.93)
Gesamtlänge Diffusionsschwänze 7.100	$l = l_n - l_p = \sqrt{\frac{2\varepsilon_0\varepsilon_r}{q}\left(\frac{1}{N_A}+\frac{1}{N_D}\right)(-U_D-U)}$	(7.94) (7.87)
Ideale Schockleysche HL-Diode	$I_D = I_0\left(e^{\frac{U_D}{U_T}} - 1\right)$	(7.103) (7.113)
Kennlinie	$I_{Du} = I_0\, e^{\frac{U_D}{U_T}} \quad (U \gg U_T) \quad I_{Sp} = -I_0 \quad (U \ll U_T)$	
Sättigungsströme Langbasisdiode ($w \gg l_p$ bzw. l_n)	$I_{0Langb.} = qAU_Tn_i^2\left(\underbrace{\frac{\mu_p}{l_pN_D}}_{\text{Löcher}} + \underbrace{\frac{\mu_n}{l_nN_A}}_{\text{Elektronen}}\right)$	(7.104)
Kurzbasisdiode ($w \ll l_p$ bzw. l_n) (Sperrstrom $\sim \frac{1}{w}$)	$I_{0Kurzb.} = qAU_Tn_i^2\left(\underbrace{\frac{\mu_p}{(l_1-l_n)N_D}}_{\text{Löcher}} + \underbrace{\frac{\mu_n}{(l_2-l_p)N_A}}_{\text{Elektronen}}\right)$	(7.104)
Temperaturabhängigkeit	$d_T = \frac{\Delta U}{\Delta T} = \alpha_T \frac{kT_0}{q} \quad \text{mit} \quad \alpha_T = \frac{E_g}{kT_0^2}$	(7.106)
Raumladungskapazität (pro Einheitsfläche) Heteroübergang (Homoübergang folgt aus $\varepsilon_{r1} = \varepsilon_{r2}$)	$c_R = \sqrt{\frac{q\varepsilon_0\varepsilon_{r1}\varepsilon_{r2}N_AN_D}{2(\varepsilon_{r1}N_D+\varepsilon_{r2}N_A)}\,\frac{1}{-U_D-U}}$	(7.158) (7.108)

Bänder PN 7.79, PIN 7.133 ($U \neq 0$!) C/U-Diagr. 7.47 Kennl.: Tunnel 7.122; Backward 7.127

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[1] Bei $N_D \gg N_A$ kann bei l_p N_D gegen $(N_D + N_A)$ gekürzt werden. Ähnliches gilt bei l_n, wenn $N_A \gg N_D$.

4 Bipolartransistor

Ersatzschaltbilder 12.6

Stromverstärkung	$A_N = \frac{I_{EC}}{I_{EE}} = \beta\gamma_E$	(9.38)
Transportfaktor	$\beta = \frac{1}{\cosh \frac{w}{L_n}} \quad (\beta \to 1 \text{ für } w \ll L_n)$	(9.38)
Emitterwirkungsgrad (vgl. Diode, I_0)	$\gamma_E = \frac{\text{Elektronenstrom}}{\text{Elektronen- und Löcherstrom}}$ $= \frac{1}{1 + \frac{\mu_{pE} N_A L_n}{\mu_{nB} N_{DE} l_{nE}} \tanh \frac{w}{L_n}}$	(9.38)
Stromverstärkung der Emitterschaltung	$B_N = \frac{A_N}{1 - A_N}$	(9.32)
Kollektorstrom	$I_C = \frac{A q D_n n_p}{w_{B_{eff}}} \quad \text{(NPN-BipT)}$ gilt nur, wenn die Rekombination in der Basis vernachlässigt werden kann	
Dichte der Minoritäten in der Basis	$n_p = n_{p0}\, e^{\frac{U_{BE}}{U_T}} \quad \text{(NPN-BipT)}$	
Steilheit	$S = \left.\frac{\Delta I_C}{\Delta U_{BE}}\right\|_{U_{CE}=\text{const.}}$	
Leitwert	$g_{CE} = \left.\frac{\Delta I_C}{\Delta U_{CE}}\right\|_{U_{BE}=\text{const.}}$	

Ladungsträgerkonzentration 9.9/9.10
Strom-Spannungskennlinien (Ebers-Moll-Gleichungen) 9.16
Minoritätenkonzentration in der Basis (Diffusionsdreiecke) 9.18/9.20
Admittanzen und Grenzfrequenzen in Kapitel 11

5 Schottky-Diode (Metall-Halbleiter-Kontakt)

Diffusionspannung $\boxed{\text{Bändermodell 7.34}}$	$U_D = \frac{\eta_n}{q} - \Phi_{Bn} \quad \text{mit} \quad \Phi_{Bn} = \Phi_M - \chi$	(7.35)
Ausdehnung der Raumladungszone	$l = \sqrt{\frac{2\varepsilon_0\varepsilon_{rHl}}{qN_D^+}(-U_D - U)} \qquad (U \leq -U_D)$	(7.45)
Kennlinie	$I_{Du} = I_0\left(e^{\frac{U}{U_T}} - 1\right) \qquad (U \geq 0)$	(7.56)
	$I_{Sp} = I_0\left(1 - e^{\frac{U}{U_T}}\right) \qquad (U \leq 0)$	(7.58)
mit Berücksichtigung des Bahnwiderstandes	$I = I_0\left(e^{\frac{U - I\,R_B}{U_T}} - 1\right) \qquad (U \geq 0)$	(7.65)
Sättigungsstrom nach der Diodentheorie (mfw $\gg$ 1)	$I_0 = F\frac{m^*}{m_0}AT^2\,e^{-\frac{q\Phi_{Bn}}{kT}}$ mit Richardson-Konst. $A = 120\frac{\mathrm{A}}{\mathrm{cm^2K^2}}$ und F dem stromdurchflossenen Querschnitt	(7.56)
nach der Diffusionstheorie (mfw $\ll$ 1)	$I_0 = Fqv_dn_R$ mit $\vec{v}_d = \mu_n\vec{E}_R$ (Randfeldstärke)	(7.58)
	und $n_R = N_L\,e^{-\frac{q\Phi_{Bn}}{kT}}$ (Elektronenkonz. am Rand)	(7.55)
Verhalten der Diode für sehr kleine Spannungen (Näherung)	$R = \frac{U_T}{I_0} \quad \text{für} \quad \lvert U\rvert \leq 10\text{mV}$	(7.65)
Temperaturabhängigkeit	$I_0(T_0{+}\Delta T) = I_0(T_0)\,e^{\alpha_T\Delta T} \quad \text{mit} \quad \alpha_T = \frac{q\Phi_{Bn}}{kT_0^2} = \frac{a}{T_0^2}$	(7.67)
Temperaturdurchgriff	$d_T = \frac{\Delta U}{\Delta T} = \alpha_T\,\frac{kT_0}{q}$	(7.71)
Raumladungskapazität (pro Einheitsfläche)	$c_R = \sqrt{\frac{q\varepsilon_0\varepsilon_{rHl}N_D^+}{2}\,\frac{1}{-U_D - U}} \qquad (U \leq -U_D)$	(7.45)

6 MIS-Struktur bzw. MOS-Struktur

Flachbandspannung	$U_{FB} = \underbrace{\Phi_{MHl}}_{\text{Differenz der Austrittsarbeit}} - \underbrace{\frac{Q_{Is}}{c_{Is}}}_{\text{negative Oxidladung}} - \underbrace{\frac{Q_{ss}}{c_{Is}}}_{\text{Oberflächenzustände}}$	(7.17)
Ausdehnung der Raumladungszone im n-Halbleiter (gleich Schottky-Kontakt, wenn $-U_D$ durch U_{FB} ersetzt wird)	$l = \sqrt{\frac{2\varepsilon_0\varepsilon_{rHl}}{qN_D^+}(U_{FB} - U)} \qquad (U \leq U_{FB})$	(7.20)
(Raumladungs-)Kapazität der Diode für die Verarmung	$c_R = \sqrt{\frac{q\varepsilon_0\varepsilon_{rHl}N_D^+}{2}\,\frac{1}{U_{FB} - U}} \qquad (U \leq U_{FB})$	(7.21)
für Anreicherung und Inversion	$c_{Is} = \frac{\varepsilon_0\varepsilon_{rIs}}{d_{Is}}$	(7.18)

Ideale MIS-Struktur 7.5ff
Reale MIS-Struktur 7.10ff
C (U, f) 7.28

7 Feldeffektransistor

Ersatzschaltbilder 12.6, 11.16
Modell 13.8
Typen-Übersicht/Gegenüberstellung 8.10
Kennlinie für MESFET und JFET 8.48
Kennlinie MOSFET 8.30ff

Beim Feldeffektransistor ist U_T die Schwellenspannung, im Gegensatz zur Temperaturspannung beim PN-Übergang oder Bipolartransistor.

Entwurfsparameter für MOSFET	$2k = \frac{\mu C_G}{L^2} = \frac{\varepsilon_{rOx}\varepsilon_0\mu}{d_{Ox}}\,\frac{w}{L}$	(8.16)
Kennlinie für kleine Aussteuerungen	$I_D = \frac{1}{R}U_{DS}$	(8.17)
Diagramm 8.18	mit $R = \frac{1}{2k\,(U_{GS} - U_T)}$ $(U_{DS} \ll U_{GS})$	
in der Sättigung	$I_{DSS} = k\,(\underbrace{U_{GS} - U_T}_{=U_{DSS}})^2$	(8.19) (8.24)
Diagramme 8.20 und 8.23	für $(U_{GS} - U_T) < U_{DS}$ z.B. Sourceschaltung	
allg. bis zum Sättigungsbereich	$I_D = 2k\,[(U_{GS} - U_T)U_{DS} - \frac{1}{2}U_{DS}^2]$	(13.8)
	für $(U_{GS} - U_T) > U_{DS}$	
Steilheit	$S = \left.\frac{dI_D}{dU_{GS}}\right\vert_{U_{DS}=\text{const.}}$	(11.12)
Gleichstromwiderstand	$R_{DS} = \frac{U_{DS}}{I_D}$	
differentieller Ausgangswiderstand	$r_{DS} = \frac{\Delta U_{DS}}{\Delta I_D}$	

8 Halleffekt

Wichtigste Meßmethode zur Bestimmung der Beweglichkeiten im Halbleiter.

Hallspanung	$U_H = -vdB_z = R_H \frac{I_x B_z}{w}$	(4.14)
Hallkonstante für n-Halbleiter im extr. Ber.	$R_H = -\frac{1}{nq}$	(4.14)
für p-Halbleiter im extr. Ber.	$R_H = +\frac{1}{pq}$	
für gemischte Halbleiter	$R_H = r \frac{p\mu_p^2 - n\mu_n^2}{q(p\mu_p + n\mu_n)^2}$	(4.15)
Widerstandseffekte 4.19–4.22	mit $r = \frac{3\pi}{8}$ wg. gekrümmter Bahnen; kann im Allgemeinen weggelassen werden ($r = 1$)	
	$\mu_n = -R_H \kappa_n$	(4.15)

9 Photoleitung

Photoleitfähigkeit	$\kappa_{Ph} = q(\mu_n\,\Delta n + \mu_p\,\Delta p) \quad \text{mit} \quad \Delta n = \Delta p$	(4.40)
stationärer Zustand	$\boxed{1} \qquad \Delta n = n_1 = \sqrt{\frac{L}{B}} \quad \left[= \sqrt{\frac{L\eta}{B}} \right]$ L = Generationsrate; B = Proportionalitätskonstante; η = Quantenwirkungsgrad $\boxed{2} \qquad \Delta n = n_1 = L\tau_n$ $\tau_n = \frac{1}{C}$ = Lebensdauer der Elektronen	(4.36) (4.36)
Anklingen nach Einschalten	$\Delta n = Lt$	(4.36)
Abklingen nach Abschalten (bei $t = t_1$)	$\boxed{1} \qquad \Delta n = \frac{\Delta n_1}{1 + \Delta n_1 B(t - t_1)}$ $\boxed{2} \qquad \Delta n = \Delta n_1\, e^{-\frac{t-t_1}{\tau_n}} \qquad \Delta n_1 = n_1$	(4.38)
Verstärkungsfaktor	$v = \frac{\Delta n v_n}{Ll} = \frac{\tau}{T_d} \quad \text{mit} \quad T_d = \frac{l}{v_n} = \frac{l^2}{\mu_n U}$	(4.42)

$\boxed{1}$	Elektronen im Leitungsband rekombinieren nur mit Löchern im Valenzband
$\boxed{2}$	viele Rekombinationsmöglichkeiten

und weiter gehts ...

Lichtabsorption im Halbleiter	Lichtintensität $I(x) = I_0\, e^{-\alpha x}$ Absorption $\sim$ Störstellenkonzentration	(5.1) (5.9)
Laser-Bedingung	$g > \frac{1}{L} \ln \frac{1}{R} + \alpha$ $R =$ Reflexionsvermögen $\approx \frac{(1-\bar{n})^2}{(1+\bar{n})^2}$ mit $\bar{n} = \sqrt{\varepsilon_r}$	(5.29) (5.26)
verfügbare Leistung einer Solarzelle $\boxed{\boxed{\text{Diagramm 7.134}}}$	$P_V = U_L I_L = F U_{L0} I_{K0} \quad U_{L0} \approx U_D \quad F =$ Füllfaktor	(7.132)
Quantenenergie	$E_{Ph} = h\,\nu = \frac{hc}{\lambda} = m_{Ph}\, c^2$ absorbierte Photonenenergie = Bandabstand E_g	(2.5) (2.130)

10 Was weiß ich

„ switched capacitor “	$R = \dfrac{1}{C f_C}$ mit $f_C \gg f_{max}$	(10.9)
$V \times B$ - Produkt	$V \times B = \dfrac{S}{c_a}$ c_a = wirksame Kapazität am Ausgang	(12.34)

Neue Norm der Gattersymbole 13.4 und 13.7

11 Vermischtes zum Fragenteil

Bandabstand

⇑	GaAs	höherer Bandabstand
‖	Si	→ geringere Eigenleitungsdichte
‖	Ge	→ höhere Betriebstemperaturen

direkter Halbleiter Valenz- und Leitungsbandmaximum liegen im Impulsraum gegenüber
→ höhere Rekombinationsrate → kürzere Trägerlebensdauer

Durchbruchmechanismen (7.115):

Lawinendurchbruch mit zunehmender Temperatur kürzere freie Weglänge
$(TK > 0; |U| > 5..6\text{V})$
→ Durchbruchspannung steigt

Zenerdurchbruch mit zunehmender Temperatur größere Tunnelwahrscheinlichkeit
$(TK < 0; |U| < 5..6\text{V})$
→ Zenerspannung sinkt

thermischer Durchbruch

Bipolartransistor

	⇑	Emitter
Dotierung	‖	Basis
	‖	Kollektor

wide-gap-Emitter (7.155, 7.137) größerer Bandabstand des Emittermaterials (PN-Heteroübergang)
→ hohe Basisdotierung möglich
→ höher Emitterwirkungsgrad
→ verbesserte Hochfrequenzeigenschaften

FET mit steigender Temperatur sinkt die Ladungsträgerbeweglichkeit und die Sättigungsgeschwindigkeit im Kanal

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